**Introduction: AMP Chain Graph**

Chain graphs (CGs):
- admit both directed and undirected edges,
- there are no partially directed cycles.

**Example:**

Chain graph models are more expressive than Markov networks and Bayesian networks.

**Main Theoretical Result 1**

- **Problem 1** (test for minimal separation) Given two non-adjacent nodes $X$ and $Y$ in an AMP chain graph $G$ and a set $Z$ that separates $X$ from $Y$, test if $Z$ is minimal i.e., no proper subset of $Z$ separates $X$ from $Y$.

- **Theorem 1.** The problem of finding a minimal separating set for $X$ and $Y$ in an AMP chain graph $G$ is equivalent to the problem of finding a minimal separating set for $X$ and $Y$ in the undirected graph $(G_{\text{ant}}(X,Y))^a$.

**Main Theoretical Result 2**

- **Problem 2** (restricted separation) Given two non-adjacent nodes $X$ and $Y$ in an AMP chain graph $G$ and a set $S$ of nodes not containing $X$ and $Y$, find a subset $Z$ of $S$ that separates $X$ from $Y$.

- **Theorem 1.** Given two nodes $X$ and $Y$ in an AMP CG $G$ and a set $S$ of nodes not containing $X$ and $Y$, there exists some subset of $S$ which separates $X$ and $Y$ if and only if the set $S' = S \cap \alpha X \cup Y$ separates $X$ and $Y$.

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**PC-like Algorithm (Peña, 2012)**

**PC-like Algorithm has two main phases:**

1. **Skeleton Recovery:** */ * for $i \leftarrow 0 \to |V_G| - 2$ do

   Select any ordered pair of nodes $u$ and $v$ in $H$ such that $u \in ad_G(v)$ and $|ad_G(u) \cup ad_G(ad_H(u))\{u,v\}| \geq i$, using order($V$); */ * if there exists $S \subseteq \{ad_H(u)\cup ad_H(ad_H(u))\}\{u,v\}$ s.t. $|S| = i$ and $u \perp \perp_p v|S$ (i.e., $u$ is independent of $v$ given $S$ in the probability distribution $p$) then
   - Set $S_u = S_u \cup S$;
   - Remove the edge $u \leftarrow v$ from $H$;

2. **Orientation phase:** */ *

   Select any ordered pair of nodes $u$ and $v$ in $H$ such that $u \in ad_H(v)$ and $|ad_H(u) \\setminus \{u,v\}| \geq i$, using order($V$); if there exists $S \subseteq (ad_H(u) \setminus \{u,v\})$ s.t. $|S| = i$ and $u \perp \perp_p v|S$ (i.e., $u$ is independent of $v$ given $S$ in the probability distribution $p$) then
   - Set $S_u = S_u \cup S$;
   - Remove the edge $u \leftarrow v$ from $H$;

**Implication:** CI tests at each level can be grouped and distributed over different cores of the computer, and the results can be integrated at the end of each level. This makes our algorithm usuable in dealing with big data via taking advantage of parallel computation.

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**LCD-AMP Algorithm**

Main idea: Learning the structure of AMP chain graphs based on the idea of decomposing the learning problem by means of separation trees into a set of smaller scale problems on its decomposed subgraphs.

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**Experimental Evaluation**

Performance metrics: TPR and Structural Hamming Distance (SHD).

**Synthetic Data:** average over 30 repetitions with 30 variables correspond to expected degree of nodes $N = 2, 3$, and the significance level $\alpha = 0.05$.

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**References**

Paper:

Code: